About the Freeware Chaos App
from Math And Such

An excellent and readable introduction to mathematical chaos is the book Chaos: Making a New Science by James Gleick.

This program is available at MathAndSuch.com. It illustrates one aspect of mathematical chaos theory.

Chaos is about the equation

$$y_{n+1} = \alpha\, y_n\, (1 - y_n)$$

where α is a constant in the range $(0,4)$.

The origin of this equation lies in population studies. The size of generation n+1 is proportional to y_n (population size in generation n) with the additional factor $1-y_n$ to account for too many resources being used by those y_n individuals.

The equation is an iteration, where $y_n \in (0,1)$. Suppose α is 2.25.

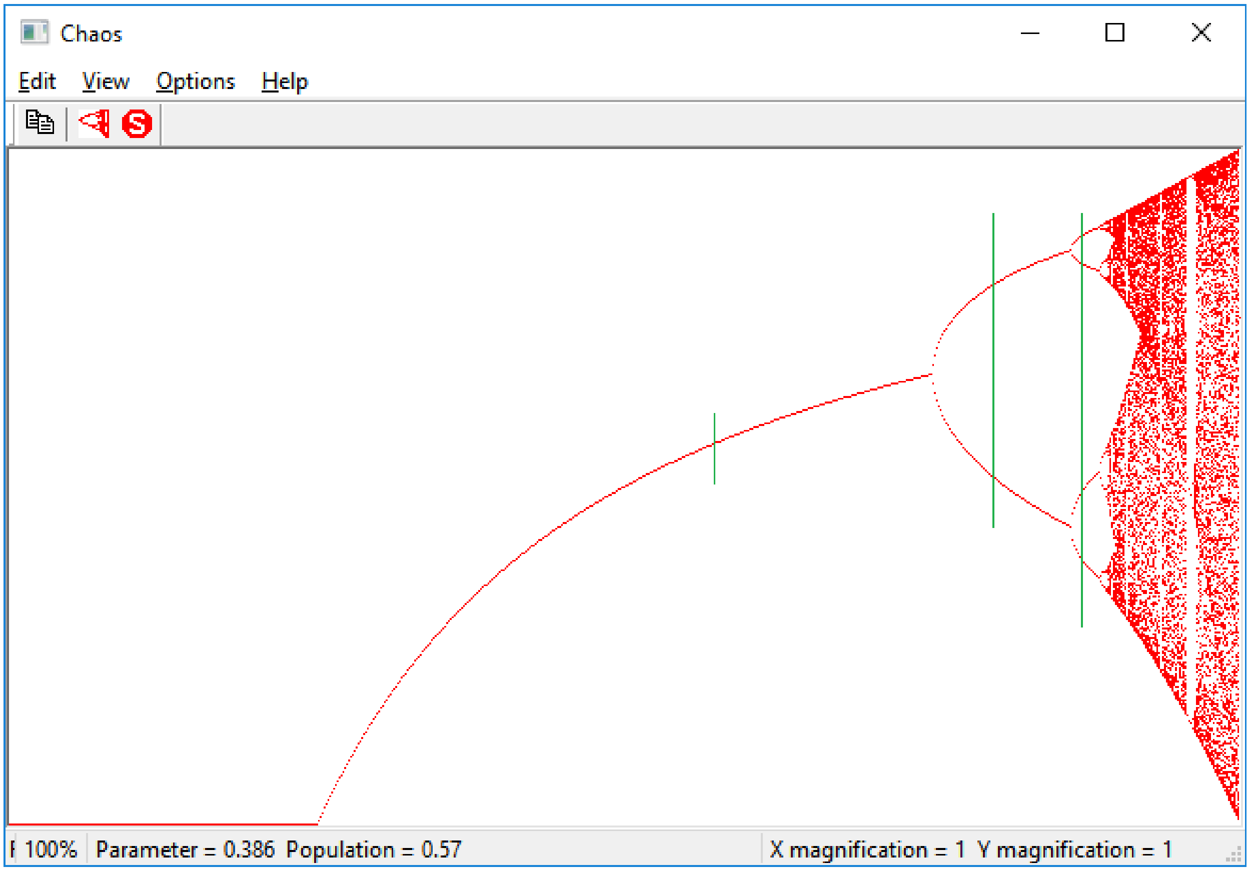
$$y_{n+1} = 2.25\, y_n\, (1 - y_n)$$

Starting with $y_1 = 0.4$, we get $y_2 = 0.54$, $y_3 = 0.559$, etc. The y values converge on 0.5555…. This convergent value depends on the value of α. The convergent is 0 if $\alpha \in [0,1]$ and is positive if $\alpha \in (1,3]$. The first green line superimposed on the screen shot below shows the convergent for α = 2.25, namely 0.5555….

If $\alpha > 3$, all bets are off and there is no single convergent. Take α = 3.2. y settles down to two alternating values: 0.513 and 0.799. As α changes so do these two convergents. The second green line in the screen shot below crosses the red graph at these two values for α = 3.2.
When α passes 3.45 we get not two values but four values that the equation cycles through. (The third green line on the graph.) Each of the two branches of the graph bifurcate at exactly the same value of α, 3.45 as

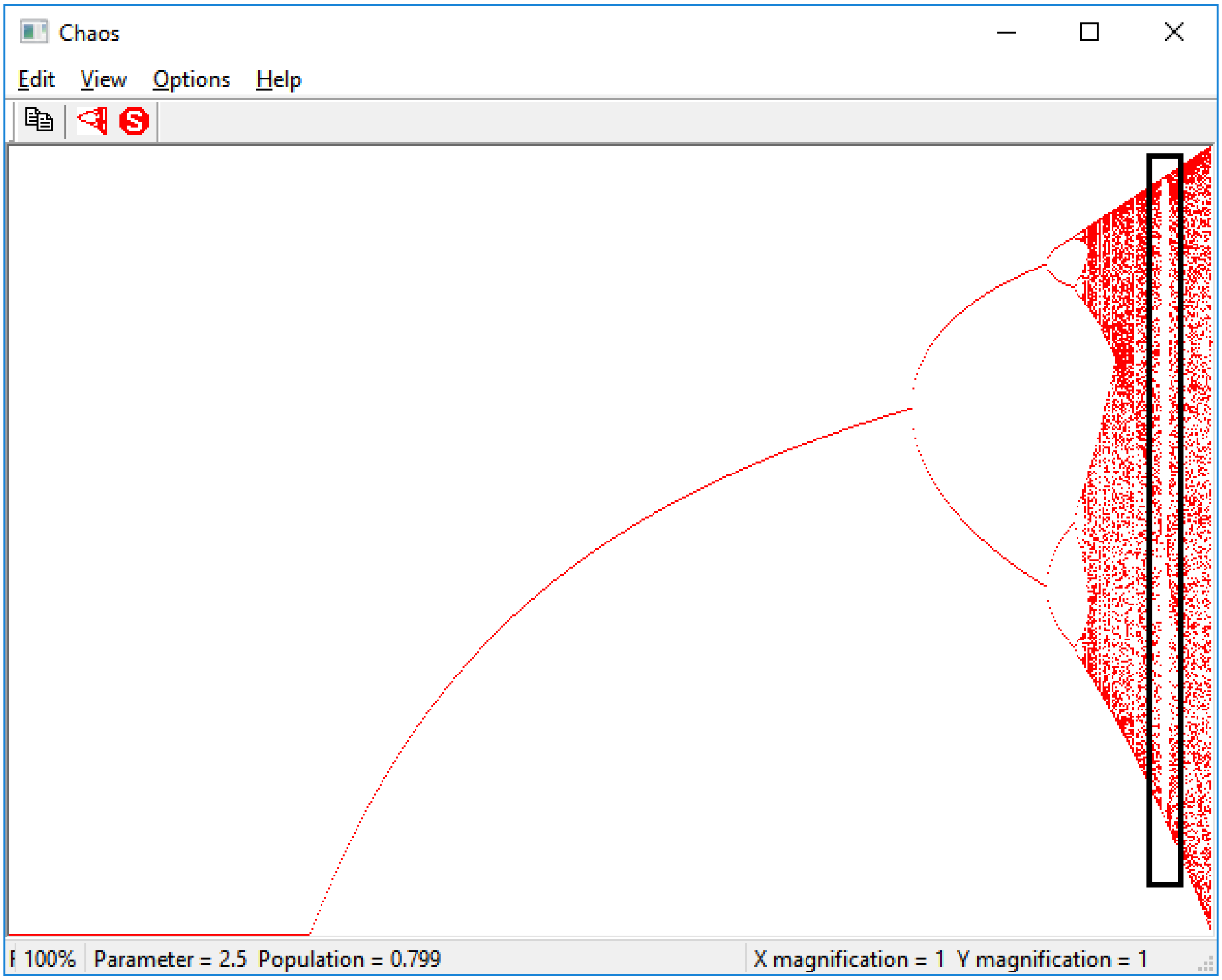
mentioned. As ɑ increases past 3.5443 there is another bifurcation of the graph and we get eight values that we cycle through.

At some point we get an infinite number of y values with no apparent sense regarding their sequence, i.e., chaotic behavior from this simple iteration.

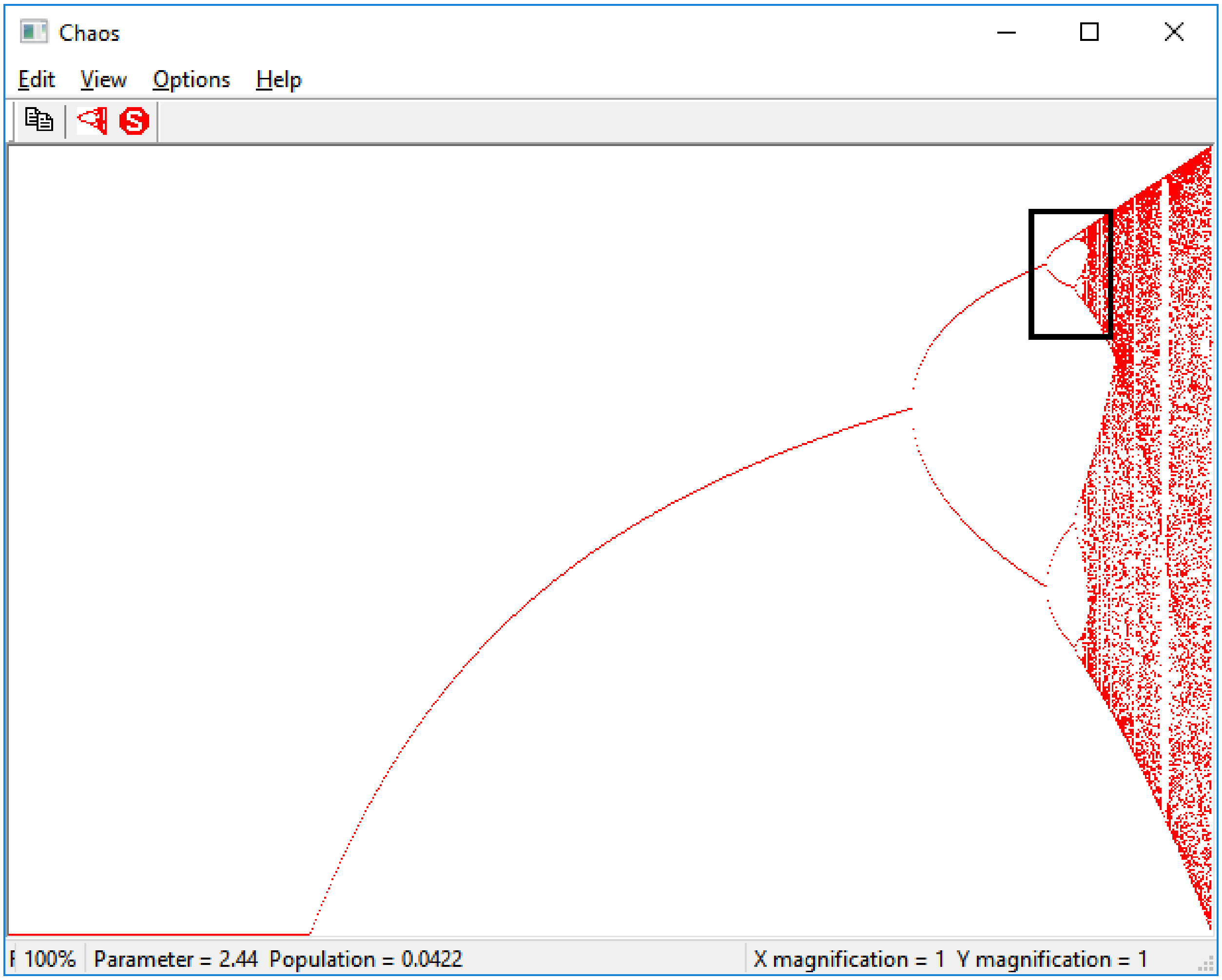


But in the realm of chaos there is some order, for as ɑ passes 3.8285 we instantly descend from an infinite number of y values to only 3 y values. This is the "thick" vertical white strip near the right edge of the graph above. Then the bifurcations set in and we get 6 values, 12 values, etc.

Zooming in shows surprising structure to the graph. Zoom in by holding down the left mouse button, moving the mouse, and releasing the button when you have defined the zoom rectangle. For example, zooming in on the vertical white strip is done with the following zoom rectangle.



Try zooming in as shown by the rectangle below and you will see that the general character of the entire graph is repeated inexactly at all levels of zooming, i.e., an infinite number of times, all a bit different from each other. Just keep zooming in and exploring.



At some point you will want to zoom in and explore the chaotic regions, which do have structure to them.

The ɑ (parameter) and y (population) of the current mouse position is displayed in the status bar. Note that the plot is of y (vertical) vs. ɑ (horizontal). The points for any ɑ are drawn after y has been highly iterated. For any ɑ the graph shows the various value(s) of y that the equation cycles through.

Play with the other controls to see what they do.

If you zoom in enough you will surpass the precision limits of floating-point numbers. If this happens, you'll know it from the graph but the program will not crash.

Chaos.exe is freeware and is unsupported. I simply thought others, particularly those who have read Gleick's book, would enjoy playing with this toy.

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