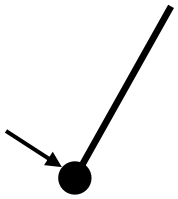
Control Theory

Control theory is the mathematics of how to apply corrections to keep a device on course, whether it be a self-driving car or a simple pendulum able only to swing back and forth in one direction. The example chosen for this vignette is the pendulum and we want it to be hanging straight down.

The pendulum is hanging outside where people can and do playfully push it. We have a way of applying a small force to the pendulum, but how much of it should we apply and in which of the two directions in order to bring it to rest vertically as quickly as possible?



Our first reaction is to simply push it toward the vertical, as shown. This has some success. But if the pendulum is already moving in that direction, we may badly overshoot. The solution is to consider both the position and the speed of the pendulum. We might want to slow the pendulum to get more quickly to a rest position hanging straight down.

In the absence of any systematic error, these two factors can solve the problem. But suppose the wind is blowing and when left hanging freely the pendulum hangs at an angle. In this case we accumulate a running average of the pendulum's position, which accumulation quickly indicates the presence of the systematic error. Given this information we can provide a restoring force to overcome the wind and bring the pendulum back to vertical.

The moral is that having a force at our command is not enough. We must understand the mathematics of how to use that force. I bet you didn't know there was a need for mathematics here. It's certainly not obvious.

Other examples include controlling electrical signals in circuits and controlling the engine gimbals of a rocket to steer the rocket on a trajectory into space.[1] Even steering a bicycle is an example of control theory.

This classical theory is called PID for Proportional (position), Integral (handling the wind), Differential (speed). Modern control theory is both more intricate and more powerful.

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[1] The wonderful movie Apollo 13 contains a scene based on reality in which Jim Lovell fires a rocket to supply the restoring force to get back to a safe trajectory, a life critical application of Control Theory done manually.