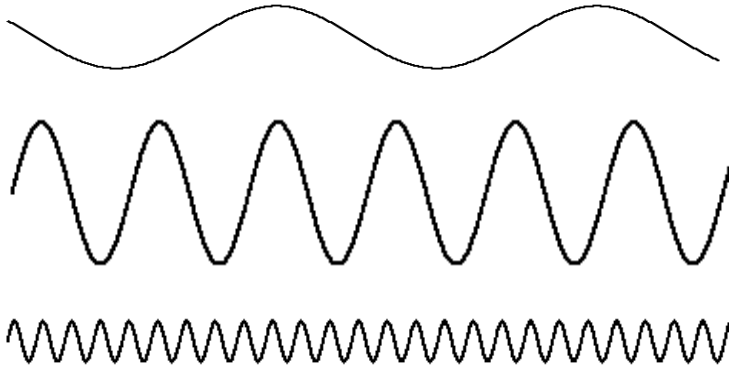


Periodicity

Joseph Fourier of the French Academy of Sciences discovered that functions can be expressed as a sum of sine waves. Not just periodic functions for which there is some intuitive basis, but any function provided one uses all frequencies. At the end of this vignette I will tell you the practical use of this.

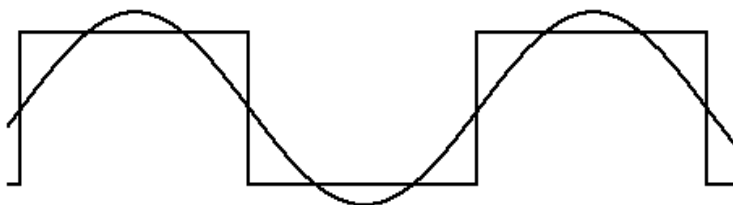
For illustration, here are three sine waves of different frequencies and amplitudes. All go forever in both directions.



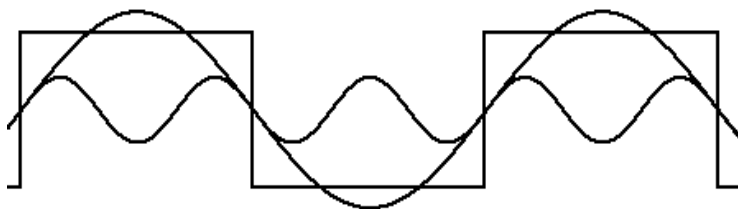
Now let's start with a simple case, a periodic square wave, periodic forever in both directions.



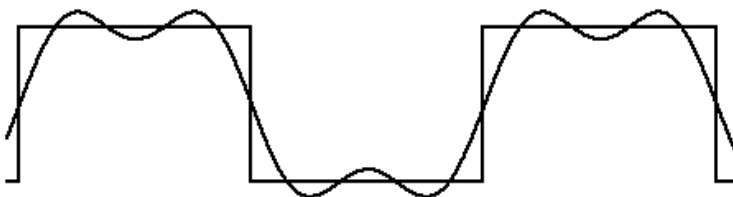
Using just a single sine wave produces a not unreasonable approximation.



Using a second sine wave corrects to some extent deficiencies in the first sine wave.



The correction becomes clearer when we sum the two sine waves, as intended.

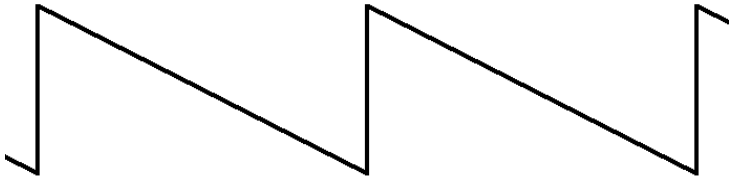


Using a few more sine waves, all having easily calculable frequencies and amplitudes, sums to the following.

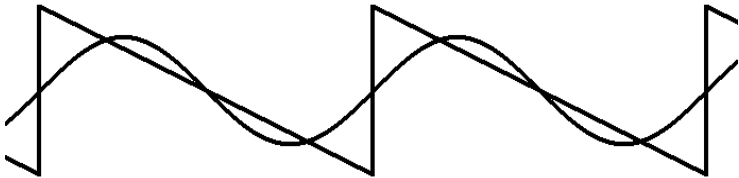


Only six sine waves add to give this superb approximation to the square wave. We can go as far as we like but the idea should be clear.

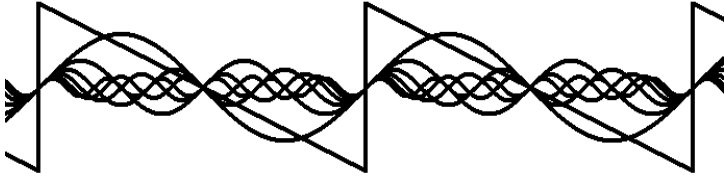
Let's look at an infinite signal with less symmetry.



Using just one sine wave is not bad but neither is it good.



Let's try the first six sine waves.



And now we sum them.



This was just part of Fourier's discovery. I wrote that all signals, even non-periodic signals, can be expressed as a sum of infinitely long sine waves. This is not at all intuitive. It is in fact counter-intuitive, particularly when the non-periodic signal is zero everywhere outside a certain range.

But it is indispensable. An electrical engineer faced with a difficult problem involving an electrical signal will decompose the signal into its infinitely long sine wave components. Then he will operate separately and easily on each component, producing different effects on each component. Finally, he reassembles the modified components to find the signal that is the answer to his problem.

Without Fourier analysis there would be little electrical engineering and no advanced electronics like cell phones. Without Fourier analysis perhaps half of all engineering could not exist and modern life would be much less modern.