Russia vs. America

In the 20th and 21st centuries, the most powerful schools[1] of mathematics have been and are those of Russia and of America. There is at least one key difference between them.

In America, given a system of difficult, nonlinear equations a mathematician will look for a linear approximation over the domain of interest. (It is impossible to find a linear approximation that works everywhere.) Americans succeed quite frequently and obtain good results.

In Russia, there is an attempt to find an exact solution to the nonlinear system. Russians succeed less often but when they do succeed, they get it exactly right everywhere.

The following is not a system of equations nor even one equation (that's good, right?), but it captures the spirit of the American vs. Russian schools. Consider a spiral such as a roll of paper towels. We want to measure the length of the roll without unwinding it. This is simple if we are always far from the center of the spiral, where "far" means far compared to the distance between consecutive windings. For a roll of paper towels this is certainly true, for there is no paper at the center.

In this case the length is approximated by

# of windings × length of middle circumference (shown dark)

Provided we don't use this near the spiral's center (should the paper be rolled in such a way), this approximation

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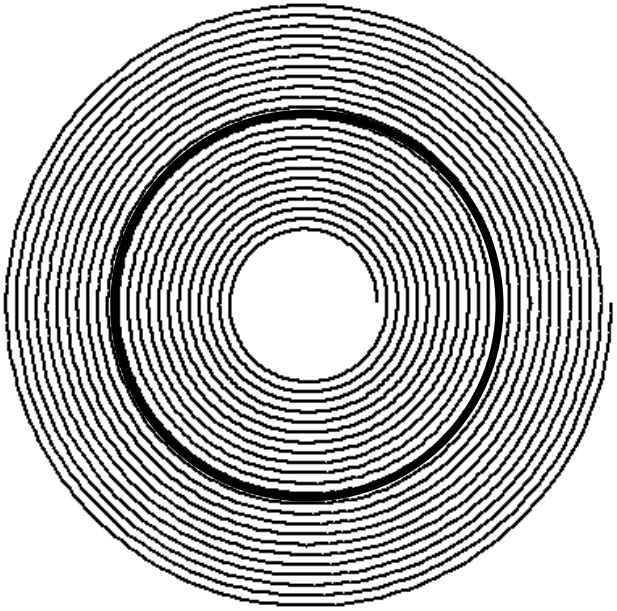
[1] From the online Merriam-Webster dictionary, a school is "persons who … follow the same intellectual methods." This is not about any educational institution but rather about a professional culture.



can only be described as superb. But suppose we have a strange roll of paper towels that does start in or near the center.

Then we need the exact formula which is the following monstrosity. Just glance at it and read on.

$$\frac{1}{2a}\left[r_2\sqrt{r_2^2+a^2} - r_1\sqrt{r_1^2+a^2} \right] + \frac{a}{2}\ln\frac{r_2+\sqrt{r_2^2+a^2}}{r_1+\sqrt{r_1^2+a^2}}$$

Amateur mathematicians may test themselves by verifying this formula does reduce to the approximate formula when r_1 and r_2, the smaller and larger radii between which we are measuring, are large compared to a. (The equation of the spiral is $r=a\theta$.)

Despite being an American I choose not to pass judgment regarding which school is mathematically better. As a practical matter let's just stick to ordinary rolls of paper towels, ok?

The earlier widespread use of computers in America may be relevant to this difference, although I don't believe it. I believe the difference is cultural.

As a minor piece of evidence to support this belief I offer an anecdote. Several decades ago, I interviewed with a company looking to hire mathematicians. While eventually declining an offer, I learned of a then recent Russian émigré who had also been offered a position. When asked about linear approximations to nonlinear systems, he had replied (and I'm trying to capture his accent), "Linear eqvations are for vomen."