

The Spherical Earth

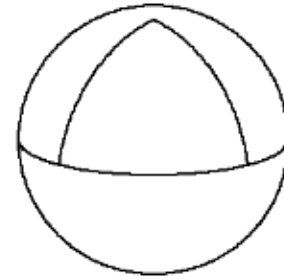
There was more to learn about the spherical earth than how to make flat maps of it. For example, given the latitudes and longitudes of two arbitrary points, what is the distance between the points as measured on a great circle? As a matter of navigation alone, finding the distance was an important issue. Yet one disadvantage of all flat maps (Mercator maps are no exception) is that most straight lines on the map are not great circles.

There is a simple formula for the distance between arbitrary points on the earth, given only their latitudes and longitudes. It is called the haversine formula should you wish to look it up, but I do not include it here.

The subject called spherical trigonometry arose from the need to measure the spherical geometry of the earth. Just as “geometry” means earth-measurement, “trigonometry” means triangle-measurement.¹ In spherical trigonometry triangles are those drawn on the earth, each side being an arc of a great circle.

One result of ball geometry allows us to determine the size of the earth. In its simplest form, one makes an equilateral triangle with each angle 90° as illustrated previously and copied below.

Multiply the length of any side of the triangle by 4 to get the circumference of the earth, from which the diameter of the earth is immediately obtained by dividing by π (use $22/7$).



In practice, there are simpler ways, although it is worth noting the original definition of the metric system. The distance from the equator to the north pole measured along the Paris meridian (it was defined by the French revolutionaries) was defined to be exactly 10,000 kilometers. An expedition was dispatched to determine the precise size of a kilometer. Fortunately for the expedition, there was no need to journey to the pole to accomplish the mission.

This leads to my last point about geometry. It is possible to determine the curvature of a surface by making distance measurements on the surface. One need not be an astronaut taking photographs of the round earth for later examination. A table of airline miles between cities shows differences between a spherical earth and a flat earth, allowing us to find the earth is spherical without leaving it and even find its radius.

This exact kind of measurement was used by a 1919 solar eclipse expedition to confirm the then hypothesized curvature of our three-dimensional universe. Welcome to the very strange world of Albert Einstein’s General Relativity.

¹ Just as a *hexagon* has six sides and a *pentagon* has five sides, so a *trigon* has three sides. The more usual word for trigon is triangle.